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# Optical Measurements of the Temperature in a Supersonic Jet

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**In this study a nonintrusive method for temperature measurement is investigated. The technique involves a two-beam schlieren system based on the measurement of angular beam deflection across an axisymmetric flow. The mean temperature is obtained by the Abel transform using the Gladstone approximation. Fluctuating temperatures are estimated by statistical processes of the beam deflections. Validation is first performed in subsonic flow, where schlieren data are compared with those obtained using classical probe. Finally, the schlieren method is successfully applied on supersonic jets approaching space launcher conditions. In the case of perfectly expanded jets, comparison between numerical and optical data shows good agreement.**

## Nomenclature

$D$ = nozzle diameter, m
$J$ = Gladstone's constant, K
$L_C$ = length of the potential core, m
$l$ = length scale, m
$M$ = Mach number
$n$ = refractive index
$P$ = pressure, Pa
$R$ = gas constant, J/mol · K
$r$ = radial component, m
$T$ = temperature, K
$V$ = jet velocity, m/s
$y$ = axial component, m
$\gamma$ = ratio of the specific heats
$\Theta$ = angle of beam deflection, rad
$\rho$ = density, kg/m$^3$
$\tau$ = fluctuation rate of temperature

*Subscripts*

$a$ = ambient condition
$i$ = stagnation condition
$j$ = perfectly expanded condition
$o$ = centerline component
$s$ = static component
$t$ = transverse component

*Superscripts*

$\bar{}$ = time average
$'$ = fluctuating component

## I. Introduction

STUDIES concerning the noise radiation associated with supersonic jets have received considerable attention in order to better understand the relations between a jet's aerodynamic characteristics and its acoustic radiation field.

In the case of space launchers, the noise radiated at liftoff is a source of significant vibration, sufficient to damage both the structure and the payload. The final objective of our research is to study the effects of temperature on the noise radiated by supersonic jets. This will be useful in improving our understanding of the physical phenomenon at the origin of acoustic variation and might provide a means for the implementation of an adapted system for sound reduction.

To analyze this effect, a method for temperature measurement must be developed and implemented for supersonic jets. However, the measurement of scalar properties, such as static temperature in supersonic flows represents a real difficulty. Probe-based techniques lead to problems of interpretation and the creation of shock-wave disturbances. Furthermore, the temperature rises so high that probe survival becomes an issue.

Thus, nonintrusive measurements, made possible by laser techniques, are attractive.[1] However, most of these techniques remain expensive and difficult. This work proposes a means of overcoming these problems. This is achieved experimentally using an optical technique inspired by the schlieren method.

Fisher and Krause[2] developed a schlieren technique based at first on the process of light absorption. In their experiments the correlation of two crossed beams' intensities gives an evaluation of the jet velocity in a subsonic flow, and later, the same technique is applied to supersonic flows.[3]

The method, based on the observation of angular laser-beam deflections, was first developed by Wilson and Damkevala.[4] The authors give an estimation of the advection velocity of structures in the mixing region and also fluctuations of density at crossed-beam point. The schlieren method is then generally used to measure density or refractive index profiles. Axial and radial deflections of a single laser beam are measured in a cold subsonic jet,[5] and by approximating the turbulent length scales fluctuations of the refractive index gradient can be thus determined. Davis developed a technique using crossed laser beams to estimate fluctuations of refractive index[6] and also refractive index profiles, convection velocities, and turbulent scales in a diffusion flame.[7]

The method developed in this paper is also based on the use of laser-beam deflections to measure values of refractive index in the flow. The remaining problem is to define relations between this parameter and the temperature. This is achieved using Gladstone's relation and the equation of state for an ideal gas.

The objective of this work is to develop and apply this new method. Furthermore, the feasibility of application to supersonic jets is examined. This temperature measurement will be useful in understanding the physical phenomenon related to temperature effect and their effects on supersonic jet noise. The optical technique

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for temperature measurement is fully described. Then, before application of the schlieren method to supersonic flow, a validation of the optical method on subsonic conditions by comparison with thermocouple data is presented.

Finally, the measurements of mean and quadratic temperature have been made on supersonic jets, and results are compared with numerical data for the case of a perfectly expanded jet.

## II. Optical Method

The angular deflection of laser beams through a medium of variable refractive index can be written as

$$\Theta = \int_0^L \frac{\boldsymbol{e}_n \cdot \boldsymbol{grad}(n)}{n} \,\mathrm{d}s \tag{1}$$

where $\Theta$ is the angular deflection over the optical path $(s)$ and $\boldsymbol{e}_n$ is the unit normal to $\mathrm{d}s$.

This angular deflection is the result of radial and axial components. In the two following sections the transverse coordinates are used for an estimation of the mean temperature in a cross section perpendicular to the flow. Moreover, the quadratic temperature distribution is obtained by statistical process on axial-beam deflections.

### A. Mean Temperature

If $r$ and $Y$ denote respectively the radial and axial distances of the laser beam from the jet centerline (Fig. 1), by Eq. (1) the transverse angular deflection is

$$\bar{\Theta}_t(Y) = Y \int_{-r}^{\infty} -\frac{1}{r}\frac{\partial \bar{n}}{\partial r}\frac{r\,\mathrm{d}r}{\sqrt{r^2 - Y^2}} \tag{2}$$

The inversion of Abel's integral (2) allows an estimation of the refractive index function $R_n(r) = -(1/r)(\partial \bar{n}/\partial r)$. In our case the mathematical inversion is carried out by means of orthogonal polynomials as proposed by Minerbo and Levy.[8]

The refractive index function can now be used to determine the local mean refractive index $\bar{n}(r)$ by integrating between any two radial distances $a$ and $r$:

$$\bar{n}(r) = \bar{n}_a - \int_a^r r \cdot R_n(r)\,\mathrm{d}r \tag{3}$$

where $\bar{n}_a$ is the ambient refractive index and $a$ is the distance where $\bar{n}_a$ can be applied.

The relation between local quantities such as temperature and refractive index can be provided by Gladstone's relation[9] and the equation of state for an ideal gas, respectively:

$$(n - 1)/\rho = \text{constant} \tag{4}$$

$$p = \rho R T \tag{5}$$

Finally, the local mean temperature is given as

$$\frac{[\bar{n}(r) - 1]\bar{T}(r)}{\bar{p}(r)} = \text{constant} \tag{6}$$

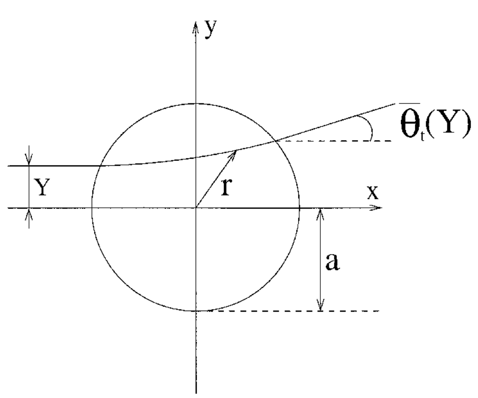


**Fig. 1 Perpendicular section to the axisymmetric flow.**

When the exit pressure of the jet is equal to the ambient pressure, the resulting mean temperature is therefore

$$\bar{T}(r) = \boldsymbol{J}/[\bar{n}(r) - 1] \tag{7}$$

where $\boldsymbol{J}$ is determined from ambient medium characteristics and the laser-beam wavelength ($\lambda = 632.8$ nm). In our case $J = 0.07946$ K.

On the contrary, when the exit pressure is different from the ambient value shock cells appear in the flow leading to variations of temperature and pressure in the downstream direction.[10] For this reason relation (6) cannot be reduced to Eq. (7). Unfortunately, a lack of information about local pressure in the jet obliges us to use the estimate

$$\bar{T}(r) \sim \boldsymbol{J}/[\bar{n}(r) - 1] \tag{8}$$

This former relation is less precise than Eq. (7) but allows nonetheless an estimation of the mean temperature in an imperfectly expanded supersonic jet.

We have defined relations allowing the estimation of mean temperature from mean transverse angular deflections. It was noticed that the inversion of Abel's integral equation is important in the computation of mean temperature. Furthermore, in the case of an imperfectly expanded jet the estimation of temperature is made difficult because the local pressure in the jet is unknown.

### B. Quadratic Temperature

The fluctuations of transverse angular deflections through the flow are integrated along the optical path. For this reason we cannot estimate local fluctuations of temperature using one single laser beam. This information can be obtained using two laser beams in a cross section perpendicular to the flow.

Consider the jet flow in the $x$ axis and two laser beams [(1) and (2)] traveling, respectively, in the $y$ and $z$ directions (Fig. 2).

Use of the relation (1) gives the axial angular deflections

$$\Theta'_{xy} = \int_C \left(\frac{\partial n}{\partial x}\right)' \mathrm{d}y \tag{9}$$

$$\Theta'_{xz}(Y) = \int_C \left(\frac{\partial n}{\partial x}\right)' \mathrm{d}z \tag{10}$$

When the two beams are crossed in a section perpendicular to the flow direction, the covariance of the axial angular deflections can be regarded as the fluctuating refractive index, responsible for the simultaneous deflection of the laser beams. Ball and Bray[11] pointed out that in anisotropic turbulence the former covariance is related to the variance of refractive index and integral scales in the jet as

$$\overline{\Theta'_{xy}\Theta'_{xz}}(Y) \sim 2\pi \overline{n'^2(x_1, y_1, z_1)}\left(l_\eta l_\zeta / l_\xi^2\right) \tag{11}$$

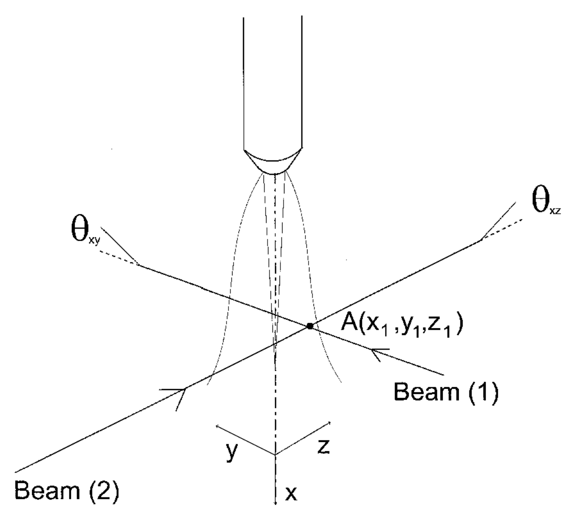


**Fig. 2 Crossed-beam schlieren system.**

where $\overline{n'^2}(x_1, y_1, z_1)$ is the quadratic refractive index measured at a point $A(x_1, y_1, z_1)$; $l_\xi$, $l_\eta$, and $l_\zeta$ are, respectively, the turbulent integral scales associated with the $x$, $y$, and $z$ directions.

Diffusion effects such as heat conduction and viscosity are slow processes. For this reason in flows at high Reynolds number these effects can reasonably be neglected. Thus, in accordance with Morfey analysis[12] it is assumed that the entropy of a particule of fluid following the motion is constant:

$$\frac{\mathrm{D}S}{\mathrm{D}t} = \frac{\partial S}{\partial t} + u_i \frac{\partial S}{\partial x_i} = 0 \tag{12}$$

As a consequence, for a perfect gas the relation between temperature and pressure is

$$T^\gamma / p^{\gamma - 1} = \text{constant} \tag{13}$$

On the other hand, it might be seen from relation (11) that the estimation of fluctuating temperature requires knowledge of the length scales. We consider that the scales in the cross section perpendicular to the jet are equivalent, according to previous work,[13] that is,

$$l_\eta \sim l_\zeta = l_r \tag{14}$$

Finally, Eqs. (4–6), (13), and (14) can be used to determine quadratic temperature

$$\overline{T'^2}(r)/\bar{T}(r)^4 \sim [(\gamma - 1)^2/2\pi \boldsymbol{J}^2](l_\xi/l_r)^2\overline{\Theta'_{xy}\Theta'_{xz}}(Y) \tag{15}$$

The variance of temperature [Eq. (15)] is thus reached from angular deflections because turbulent scales are well estimated.

## III. Validation of the Method

A set of tests are carried out in order to validate the optical method before application to supersonic jets. Many validations and experiments have been made in the past using the schlieren system to obtain refractive index values in axisymmetric flows. The emphasis here is on temperature estimation via measurements of refractive index.

The experiments described here are conducted on a jet with nozzle diameter of 30 mm. The airflow is at first generated by a fan and is then heated by a resistance. The velocity and the temperature of the jet are equal to 50 m/s and 750 K, respectively, on the centerline at the exit plane. By strioscopy observation the length of the potential core is estimated as three diameters.

In addition to the flow generator, two photosensitive diodes measure the angular deflections of He–Ne laser beams. The accuracy of the experimental results is determined by studying the influence of the angular deflections on the mean temperature calculation. It appears that the inaccuracy of 5% in the measurement of the beam deflection leads to an inaccuracy of 15% in the estimation of the temperature. One laser is fixed relative to the centerline, whereas the other can be displaced to a number of different axial positions. At a fixed position both radial and axial angular deflections are measured simultaneously.

In addition, a comparison between the mean temperature obtained using the optical method and those measured by a probe is possible. The fluctuating temperature rate is compared with experimental values measured by Abramovich[14] in similar jet conditions.

The mean temperatures from the optical method and thermocouple are compared and show good agreement (Fig. 3).

It is clear that centerline temperature does not agree exactly for any two methods, but the observed differences do not exceed 10%. Moreover, schlieren data show overestimated and underestimated temperatures as compared to probe values. As can be seen from Abel's integral inversion study, when we vary slightly angular deflection profiles, the temperature gradient changes a little, while the centerline temperature remains the same.

For the estimation of the fluctuating temperatures [Eq. (15)], we have defined $l_\xi = 5l_r$, as measured by Lau[15] in similar jet conditions. Thus, the observed fluctuating temperature levels show, as expected, that the maximum occurs in the middle of the mixing layer (Fig. 4). Also, as a result of the growing mixing layer it appears that the

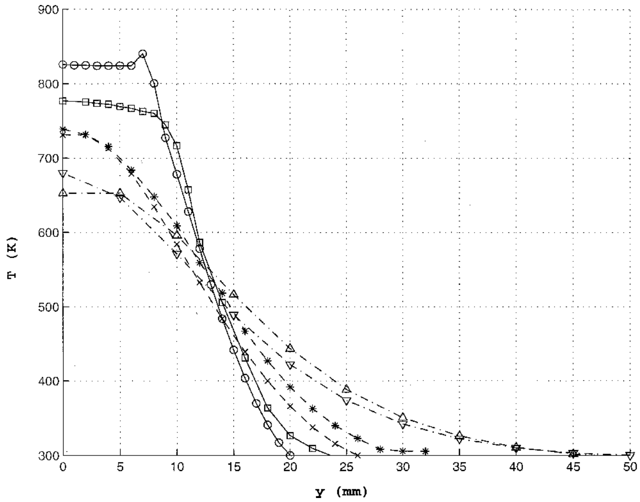


**Fig. 3 Mean temperatures measured by optical method and thermocouple. Optical data, *X/D*: ○, 0.5; ×, 1.5; and △, 4. Thermocouple data, *X/D*: □, 0.5; ∗, 1.5; and ▽, 4.**

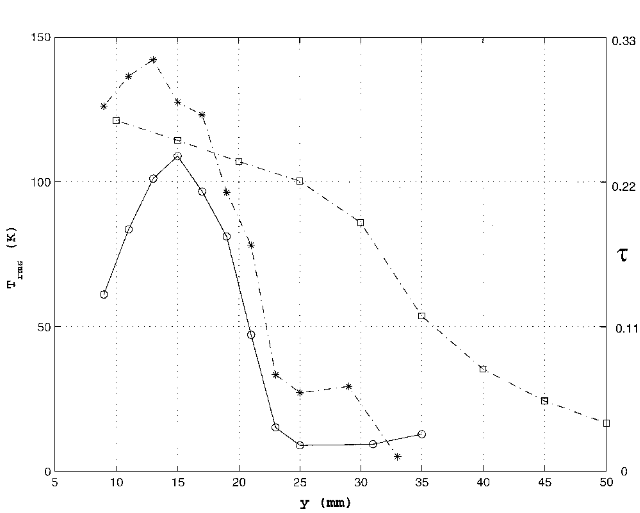


**Fig. 4 Quadratic temperatures measured by optical method. *X/D*: ○, 0.5; ∗, 1.5; and □, 4.**

farther downstream the measurement position, then the closer the peak value to the centerline is, and the more the profile spreads out.

The flatness of the profile at $X = 4D$ is expected, considering that at this position the measurements are made downstream of the potential core.

Results obtained are qualitatively correct. However, the fluctuating temperatures estimated at $X = 0.5D$ and $1.5D$ downstream of the nozzle exit differ by 20% (respectively, $T_{\mathrm{rms}} \sim 100$ and 140 K). This difference can be explained as the influence of the mean temperature value at the radial position where the maximum occurs ($r_{\max}$). Indeed, we observe $\bar{T}(r) = 450$ K ($X = 0.5D$, $r_{\max} = 15$ mm) and $\bar{T}(r) = 500$ K ($X = 1.5D$, $r_{\max} = 12$ mm); this influences the estimation of the maximum fluctuating temperature.

The mean value of the different maximum peaks, $T_{\mathrm{rms}} \sim 120$ K, corresponds to a fluctuation rate ($\tau = T_{\mathrm{rms}}/[T_o - T_a]$, where $T_o$ and $T_a$ correspond respectively to the centerline and the ambient temperature) equal to 0.25, in agreement with Abramovich's results.[14] On the other hand, the intensity of turbulence based on definition $T_{\mathrm{rms}}/T_o$ is equal to 0.16. This value is similar to the intensity of turbulence $(\overline{V'^2})^{1/2}/V_o = 0.17$ at $r_{\max} = 13.5$ mm of similar unheated jet, where $V_o$ corresponds to the centerline velocity, measured using laser Doppler velocimetry (LDV) in the same apparatus.[16]

The results obtained using the schlieren method are encouraging for application in supersonic conditions. However, the extreme acoustic environment and thermal radiation encountered in this experimental facility can result in the deterioration of the quality of angular deflection measurements.

## IV. Temperature Measurements in Supersonic Jets

The optical method has been validated in subsonic flow. The application to a supersonic jet remains an important stage in the feasibility assessment of temperature measurements in such conditions. Our results are compared with numerical data computed with similar geometry.

### A. Jet Test Conditions

Experiments were carried out at the MARTEL facility,[17] which operates with a convergent-divergent nozzle with a 50-mm exit diameter. This geometry is designed to provide a perfectly expanded jet without shock cells for stagnation pressure $P_i = 30$ bar and stagnation temperature $T_i = 1900$ K ( jet 1 in Table 1). When stagnation conditions differ from the former values, jets are imperfectly expanded; here jet 2 is underexpanded. Flow across shock cells that appear in supersonic regions of the jet is subjected to temperature (and also pressure) variations.[10] As a consequence, centerline temperature estimations are expected to show high variations compared to those in jet 1.

The experimental procedure remains identical to Sec. III. To optimize the duration of the experiment, the mobile laser beam makes 14 stops across the jet over 3 s.

### B. Mean Temperature

In the case of a perfectly expanded jet (Fig. 5), we observe an axial temperature similar to the theoretical value, $T \sim 860$ K (Table 1), irrelevant of position. Moreover, the farther the measurements from the nozzle exit, the more the temperature profile spreads out. This is because of the extent of the mixing layer.

Jet 2 presents the same results concerning the profiles, but centerline temperatures are different from the values proposed by the theory, which does not consider the presence of shock cells. As we just said, this presence leads to centerline temperature variations, which is observed here (Fig. 6).

### C. Quadratic Temperature

For the estimation of the fluctuating temperature, we have defined as in the preceding sections $l_\xi = 5l_r$ in relation (15) for the two jets. This hypothesis is open to criticism because this former rate was measured in subsonic jet. Moreover, turbulent scales, and consequently $l_\xi / l_r$, depend on temperature[13] and should be adapted in accordance (i.e., $l_\xi / l_r$ would be greater in jet 2 than in jet 1). However, no previous work estimates this rate for supersonic jets or its evolution with temperature. We decided intentionally to keep the said value bearing in mind that quadratic temperature estimations are underestimated and all of the more for jet 2.

**Table 1 Jet test conditions ($L_c/D$, length of the potential core)**

| Jet | $P_i$, bar | $T_i$, K | $T_s$, K | $V_j$, m/s | $L_c/D$ |
|---|---|---|---|---|---|
| 1 | 30 | 1900 | 862 | 1700 | 14 |
| 2 | 17 | 2100 | 1100 | 1700 | 11 |

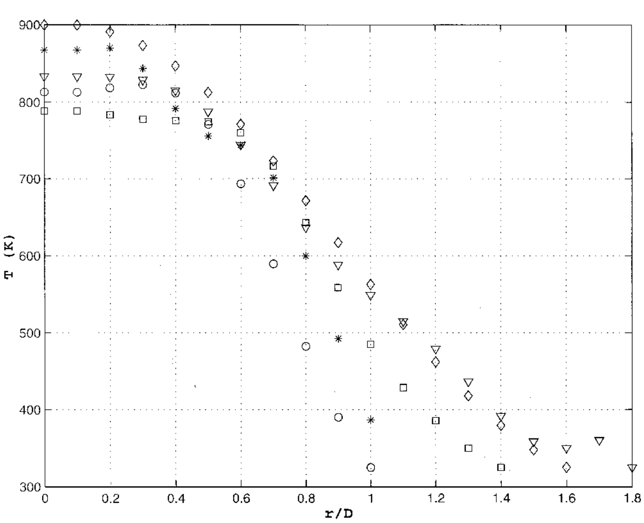


**Fig. 5 Mean temperature measured in jet 1 ($V_j$ = 1700 m/s, and $T_s$ = 862 K). *X/D*: ○, 3; ∗, 4; □, 6; ◇, 8; and ▽, 10.**

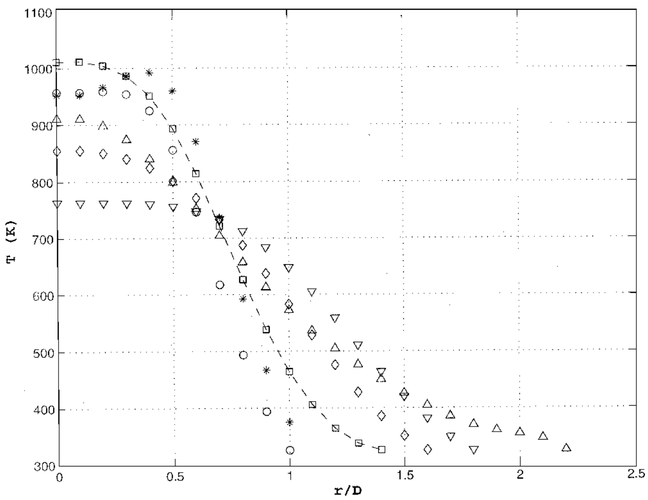


**Fig. 6 Mean temperature measured in jet 2 ($V_j$ = 1700 m/s, and $T_s$ = 1110 K). *X/D*: ○, 3; ∗, 4; □, 6; ◇, 8; ▽, 10; and △, 12.**

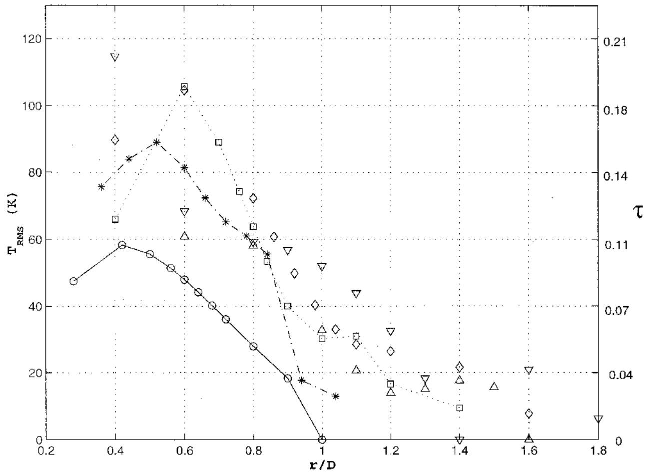


**Fig. 7 Quadratic temperatures measured in jet 1 ($V_j$ = 1700 m/s and $T_s$ = 862 K). *X/D*: ○, 3; ∗, 4; ◇, 8; ▽, 10; and △, 12.**

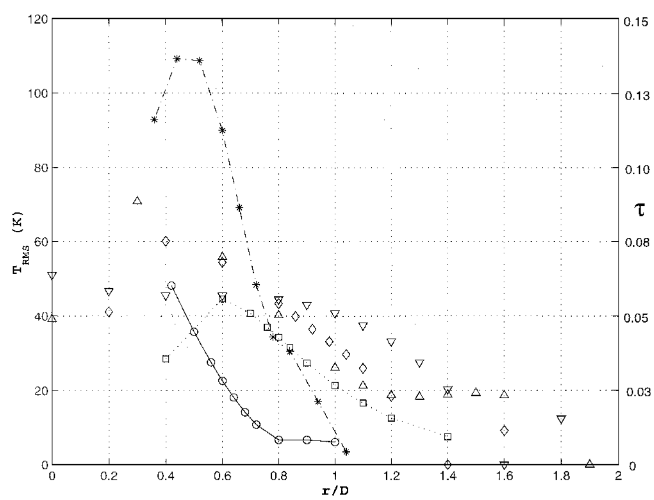


**Fig. 8 Quadratic temperatures measured in jet 2 ($V_j$ = 1700 m/s and $T_s$ = 1100 K). *X/D*: ○, 3; ∗, 4; □, 6; ◇, 8; ▽, 10; and △, 12.**

As expected, the fluctuating temperature distributions, shown in Figs. 7 and 8 for several positions, show that a peak appears in the middle of the mixing layer corresponding to a maximum of turbulence. Nevertheless, the radius corresponding to the maximum of $T_{\mathrm{rms}}$ is increasing with $X/D$, which is the opposite in subsonic condition (Fig. 4). This may be due to the jet instability.

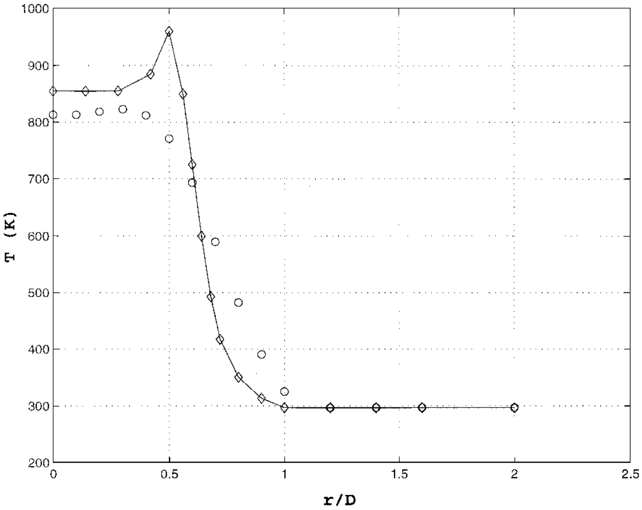


**Fig. 9 Comparison of optical mean temperatures (○) with numerical predictions (◇) in jet 1 at $X$ = 3$D$.**

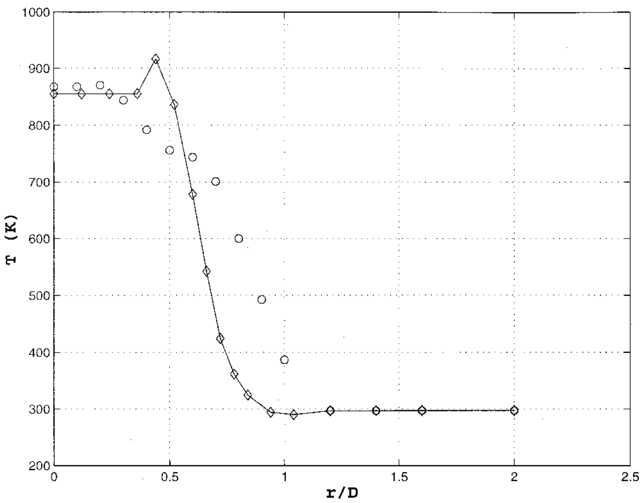


**Fig. 10 Comparison of optical mean temperatures (○) with numerical predictions (◇) in jet 1 at $X$ = 4$D$.**

In the case of jet 1, $T_{\rm rms} \sim 90$ K (i.e., $\tau \sim 0.16$) for positions from $4D$ to $12D$. For the imperfectly expanded jet, $T_{\rm rms}$ profiles seem to be more disrupted, and $T_{\rm rms} \sim 60$–80 K (i.e., $\tau < 0.1$). These two observations reveal in fact shock cell influences in measurements. When the jet is perfectly expanded, the temperature and the associated fluctuations show steady profiles. This is no more the case in the presence of shock cells.

A comparison between fluctuation levels from jets 1 and 2 shows surprisingly lower values in the former jet, whereas the mean associated temperatures are higher ($T_s = 1100$ K, $T_{\rm rms} \sim 60$–80 K for jet 2 and $T_s = 862$ K, $T_{\rm rms} \sim 90$ K for jet 1). In fact, fluctuations of temperature are comparatively underestimated in the hotter jet. As we just said, this is because the identical turbulent scale ratio was used for jets 1 and 2, which is not the case physically. An adapted value, higher than that used here, would increase the fluctuations of temperature in jet 2.

### D. Comparison of Experimental Results with Model Predictions

A comparison between temperatures from a numerical approach[18] and those measured is presented in this section. The numerical code (AMLJET) is based on the resolution of the Reynolds-averaged Navier–Stokes equations. A compressible correction model is used in conjunction with a classical $k\epsilon$ turbulent closure. Furthermore, these computations have been carried out using geometrical characteristics of the nozzle used in the experiments.

The result of this mean temperature comparison is shown in Figs. 9–11. Although agreement is reasonable, there is a definite tendency for the optical method to underestimate the gradient of temperature. This is inherent in the inversion of Abel's integral because the mathematical method[8] has real difficulty in estimating a step-shaped temperature profile from a distribution of angular deflections. Nevertheless, the two methods give approximately the same temperature on the centerline.

We notice a peak in numerical data that occurs at the edge of the potential core caused by friction between the two layers. This is not observed in schlieren measurements.

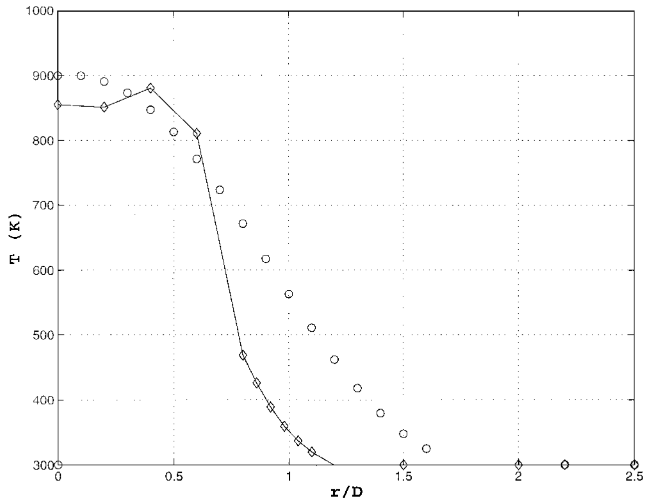


**Fig. 11 Comparison of optical mean temperatures (○) with numerical predictions (◇) in jet 1 at $X$ = 8$D$.**

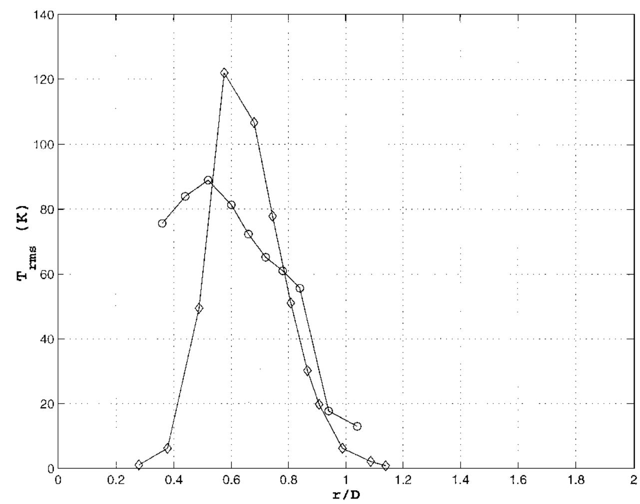


**Fig. 12 Comparison of optical quadratic temperatures (○) with numerical predictions (◇) in jet 1 at $X$ = 4$D$.**

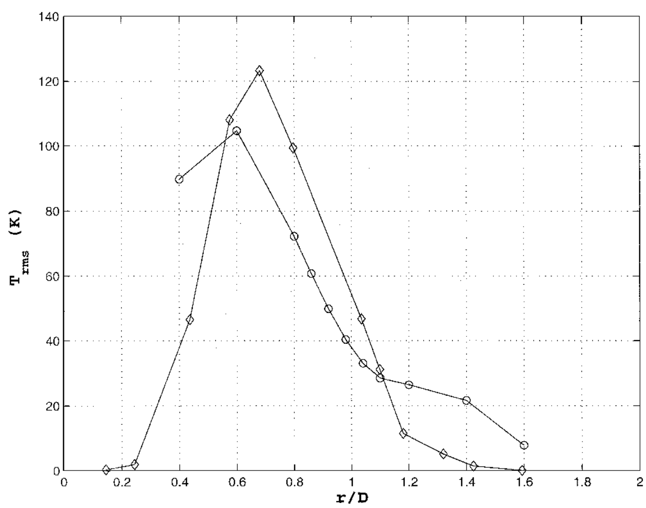


**Fig. 13 Comparison of optical quadratic temperatures (○) with numerical predictions (◇) in jet 1 at $X$ = 8$D$.**

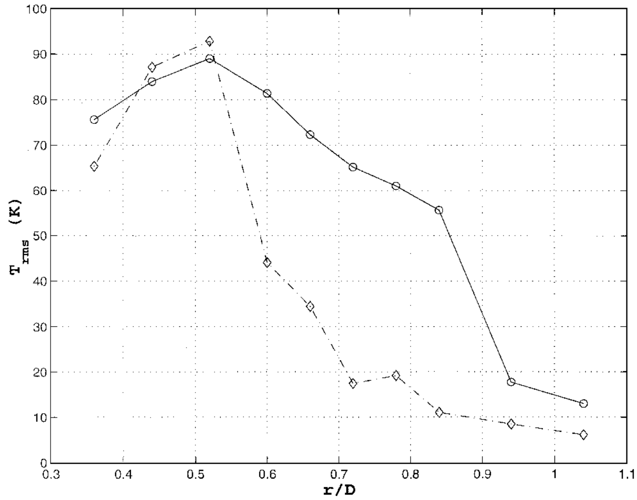


**Fig. 14 Effects of numerical mean temperatures (○) on the estimation of quadratic temperatures in jet 1 at $X = 4D$ instead of optical mean temperature (◇).**

The comparison of fluctuating temperature is shown in Figs. 12 and 13. In the two cases a maximum of fluctuation levels occurs in the middle of the shear layer. It appears that the low gradient of temperature measured with the optical method yields a broad profile of fluctuating temperature, which is not the case in the numerical data. To demonstrate this effect, numerical temperatures are used for the computation of fluctuating temperature [relation (15)] instead of those obtained with the optical method. It turns out that the value of the peak is not greatly affected, whereas the profile becomes narrow (Fig. 14).

## V. Conclusions

In this paper an optical method for temperature measurements based on the observation of angular beam deflections was investigated. This study is of interest in aeroacoustics for improved understanding of the part played by temperature in supersonic jet noise.

In a preliminary study comparisons of the schlieren temperature measurements with those obtained from probe techniques in subsonic flow showed good agreement. The maximum fluctuation rate, observed in the mixing region, remains in good agreement with published measurements.[14] It appears also that the intensity of turbulence $[(\overline{V'^2})^{1/2}/V_o]$ is similar.

The application of the method to supersonic flow leads to good results, especially when the jet is perfectly expanded. On the contrary, when the jet is imperfectly expanded the presence of shock cells leads to some uncertainty in the estimation of the temperature. This could be solved with the information of local pressure; however, this parameter is as difficult to measure as the temperature.

The estimation of quadratic temperature strongly depends on the quality of the values of length scales. Although many experiments were easily carried out in subsonic flow for various jet conditions, estimation is still difficult in supersonic conditions. For this reason, the measured profiles of quadratic temperature were qualitatively good but remain quantitatively inaccurate. It is planned to use LDV to deduce length scales in a similar supersonic jet. This would improve the measurements and also our understanding of the role of the temperature in supersonic jet noise.

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